Roll No. ....

BCA–303(O)

B. C. A. (Third Semester) EXAMINATION, Dec., 2013

(Old Course)

Paper Third

MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE – III

Time : Three Hours]

[*Maximum Marks : 75*

[*Minimum Pass Marks : 26*

Note : Section A is compulsory. Attempt *seven* questions out of ten questions from Section B and *one* question from Section C.

Section – A

(Numerical/Analytical/Problematic Questions)

1. (a) Find the rank of the matrix : 4

$$\begin{bmatrix} 1 & 3 & -4 \\ -1 & -3 & 4 \\ 2 & 6 & -8 \end{bmatrix}$$

(b) Test the convergence of the series whose nth term is : 3

$$U_n = \frac{n^n}{(1+n)^n}$$

2. (a) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$, prove that : 4

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2\sin\theta$$

(b) Find the maximum/minimum values of : 4

$$f(x, y) = 2 + 2x + 2y - x^2 - y^2$$

Section – B 6 each

(Short Answer Type Questions)

3. Determine the nature of the series :

$$x + \frac{3}{5}x^2 + \frac{8}{10}x^3 + \frac{15}{17}x^4 + \ldots\ldots + \frac{n^2-1}{n^2+1}x^n + \ldots\ldots x > 0$$

4. Test the convergence or divergence of the series whose general term is $U_n = \sqrt{n^2+1} - n$.

5. Expand log sec x by Maclaurin's theorem.

6. If $u = (1 - 2xy + y^2)^{-1/2}$, prove that :

$$x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2u^3$$

7. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that :

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$$

8. If $u = \frac{x}{y-z}, v = \frac{y}{z-x}, w = \frac{z}{x-y}$ then show that :

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = 0$$

9. Find the value of the Jacobian $\frac{\partial(u, v)}{\partial(r, \theta)}$ where :

$$u = x^2 - y^2, \quad v = 2xy \text{ and}$$
$$x = r\cos\theta, \quad y = r\sin\theta$$

10. Show that the function :

$$f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$$

is maximum at (– 7, – 7) and minimum at (3, 3).

11. Find the characteristic roots (or eigen values) of the matrix :

$$\begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$$

12. Solve the following system of equations by using Cramer's rule :

$$2x - 3y + 4z = -9$$
$$-3x + 4y + 2z = -12$$
$$4x - 2y - 3z = -3$$

Section – C 18

(Long Answer Type Questions)

13. State and prove the Euler's theorem for Homogeneous function of degree n.

Using Euler's theorem prove that, if :

$$u = \sec^{-1}\left(\frac{x^3 - y^3}{x + y}\right),$$

show that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u,$$

then evaluate :

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$$

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14. State and prove Cayley-Hamilton theorem, also verify the theorem for the matrix :

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

15. Using Lagrange's method of undetermined multipliers, find the maximum value of $u = x^p y^q z^r$ when the variables x, y, z are subject to the condition :

$$ax + by + cz = p + q + r$$